

# The Role of Learning in Accounting for the Rise in Consumer Unsecured Debt and Bankruptcies

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## Abstract

Between 1984 and 1998 the unsecured debt-to-income ratio of U.S. households increased from 4.9% to 9.1%, while consumer bankruptcies rose from 2.0 to 8.3 filings per 1,000 adults. This paper offers an explanation for these facts based on the premise that this period, commonly referred to as the Great Moderation, exhibited decreased economic volatility. In particular, we construct a model of optimal default in which both households and creditors learn about economic fundamentals in the presence of aggregate uncertainty. We prove that an increase in the debt-to-income ratio of households in our model is the natural result of realizing a sequence of aggregate shocks similar to those experienced during the Great Moderation. A calibrated version of our model is able to account for 58% of the rise in the unsecured consumer debt-to-income ratio and 64% of the increase in the bankruptcy filing rate over this period. In addition, we demonstrate that close alternatives to our model which abstract from learning are unable to explain the data.

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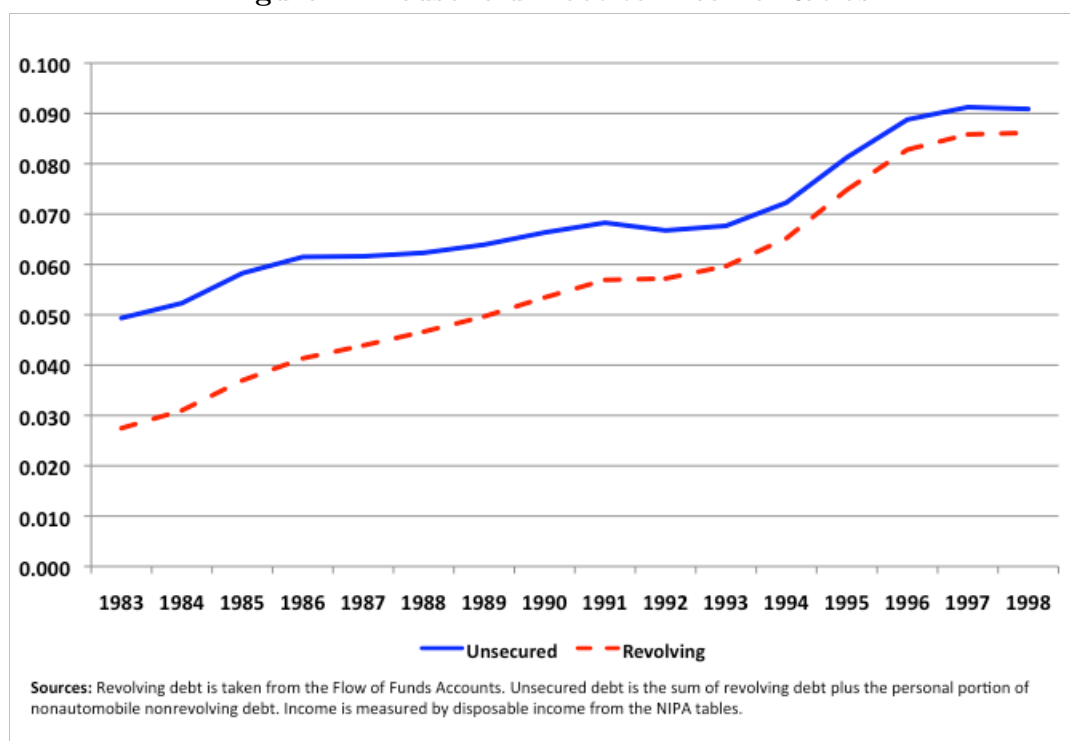
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# 1 Introduction

Between 1984 and 1998 the U.S. economy experienced an explosive rise in consumer unsecured debt and bankruptcy filings. Figure 1 depicts consumer unsecured and revolving debt as a fraction of disposable income, while Figure 2 plots annual consumer bankruptcy filings relative to the adult population.<sup>1</sup> The unsecured debt-to-income ratio for U.S. households almost doubled over this period, increasing from 4.9% in 1984 to 9.1% in 1998. Perhaps more surprisingly, after remaining remarkably stable for nearly twenty-five years, consumer bankruptcies more than quadrupled, rising from 2.0 per 1,000 adults in 1984 to 8.3 in 1998.

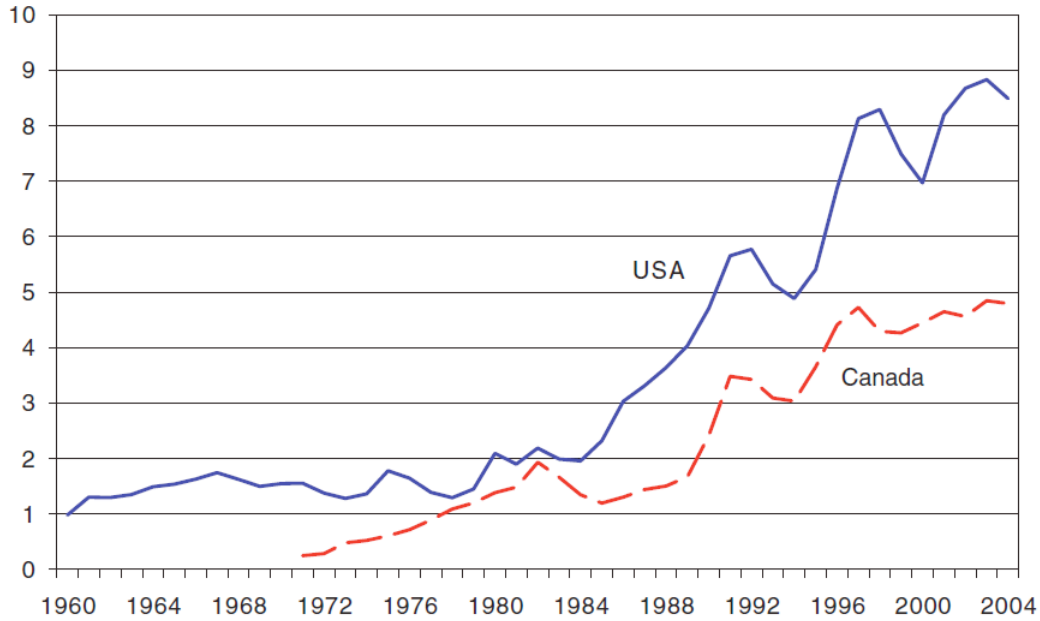
**Figure 1. Household Debt-to-Income Ratios**



Many explanations for the rise in the debt-to-income ratio and bankruptcy filings have been proposed in the empirical literature. Boyes and Faith (1986) and Shepard (1984), for example, argue that changes in the U.S. consumer bankruptcy code made

<sup>1</sup>Unsecured debt is defined as the sum of revolving debt and the personal loan portion of nonautomobile nonrevolving debt as in Livshits, MacGee, and Tertilt (2010).

**Figure 2. Total Consumer Bankruptcies per 1,000 Adults**



**Consumer Bankruptcies per 1,000 of 18 – 64 Year-Olds**  
(Source: Livshits, MacGee and Tertilt (2010))

declaring bankruptcy more attractive to potential filers. Buckley and Brinig (1998), Gross and Souleles (2002), and Fay, Hurst, and White (2002), on the other hand, contend that the rise in defaults was primarily a result of a decline in the cost of filing for bankruptcy, either non-pecuniary or pecuniary in nature. Hacker (2006) and Barron, Elliehausen, and Staten (2000) argue that an increase in income volatility led more households into financial trouble; Warren and Tyagi (2003) highlight the role of greater idiosyncratic expense risk; and Barron and Staten (2003) cite credit market innovations which reduced the transaction costs associated with issuing debt. Using an equilibrium model of optimal consumer default, Livshits, MacGee, and Tertilt (2010) evaluate the ability of each of these theories to quantitatively account for this experience. They conclude that a decline in the cost of filing for bankruptcy – broadly defined – along with a simultaneous reduction in transaction costs is the

most plausible explanation for these trends.<sup>2</sup>

In this paper we propose an alternative explanation for the rise in consumer debt and bankruptcies based on the premise that this period was a time of great change in terms of economic volatility. Starting from the canonical model of optimal default, we introduce aggregate uncertainty and posit a dynamic learning process for both households and creditors.<sup>3</sup> Agents in our model adjust their expectations in response to the realized sequence of aggregate shocks and economic fundamentals. Taking their state-dependent endowment process as given, households form beliefs about the probability of transitioning between aggregate states and use these beliefs to construct their optimal decision rules.<sup>4</sup> Creditors form expectations about household default probabilities by observing the history of all default decisions, conditional on the aggregate state, the household's previous endowment, and the loan size. Creditors then use these expectations to compute the default premium that they must charge for each loan contract.

As the central result of this paper, we find that a simultaneous rise in the consumer debt-to-income ratio and the bankruptcy filing rate – on the order of that observed in the data – is the natural response of our economy to a sequence of favorable aggregate shocks.<sup>5</sup> The intuition for this result is relatively straightforward: Realizing a string of favorable aggregate shocks leads households to become more optimistic and discount the probability of transitioning from expansion to recession. This lower perception of endowment uncertainty reduces households' precautionary savings motive, leading to increased borrowing. Moreover, for any given endowment and loan size, the probability of default is increasing in the volatility and decreasing in the mean of the endowment process. Hence, realizing a sequence of favorable aggregate shocks results in a lower than anticipated default rate for any given debt contract. In response, creditors revise downward their expectations about default probabilities and reduce the default premium charged on debt contracts. This leads

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<sup>2</sup>In particular, Livshits et al. (2010) find that in order to generate the observed rise in debt and default the social stigma cost of filing for bankruptcy in 1984 would need to be equivalent to an 11.5% reduction in a household's lifetime consumption stream and the proportional transaction cost of issuing debt would need to have fallen by roughly 4 percentage points over the period.

<sup>3</sup>The canonical model that we have in mind is that of Eaton and Gersovitz (1981) which has been used extensively in the literature to study both consumer and sovereign default.

<sup>4</sup>Storesletten, Telmer, and Yaron (2004) use data from the PSID to argue that uninsurable idiosyncratic income shocks have a larger variance during recessions than expansions. We exploit this observation, along with the related fact that mean income is lower during recessions than expansions, to specify a state-dependent endowment process for households.

<sup>5</sup>We will define precisely what is meant by 'favorable aggregate shocks' in what follows, but for now one should think about these states as those for which the households' endowment process has a relatively low variance and a high mean.

to lower interest rates which induces households to borrow more, further raising the debt-to-income ratio. Finally, since the likelihood that a household will default is increasing in their debt level, more borrowing leads to a higher incidence of default.<sup>6</sup>

In a calibrated version of our model, we find that this learning-driven credit channel can account for 58% of the rise in the unsecured consumer debt-to-income ratio and 64% of the increase in the bankruptcy filing rate between 1984 and 1998. We demonstrate that a quantitatively significant rise in both of these statistics is a robust result across different parameterizations of the learning process. While our model is able to closely replicate this experience, we show that models which abstract from learning cannot. In particular, we analyze two alternative models, each of which is consistent with a widely held view of the Great Moderation based on the rational expectations hypothesis, and illustrate that neither is able to generate a simultaneous rise in debt-to-income and bankruptcy rates on the order of that found in the data.

We view our study of this new and novel mechanism as complementary to the work of Livshits et al. (2010). Although learning can account for much of the rise in consumer debt and default over this period, we believe that the remaining portion could easily be accounted for by a reduction in bankruptcy filing costs and lower transaction costs as emphasized in the work of those authors.

The remainder of this paper proceeds as follows. In Section 2 we discuss the role of learning and argue that explicitly modeling the dynamic learning process of agents makes sense in the context of the Great Moderation. Section 3 introduces our full model that we use to generate our quantitative results later in the paper. Section 4 presents analytical results along with a simplified version of the model that provides intuition for our quantitative results. In Section 5 we formally calibrate our model and quantify how much of the increase in consumer debt and bankruptcy over the Great Moderation can be explained by learning. We also present robustness checks and consider the ability of several related specifications that ignore learning to account for these facts. Finally, Section 6 concludes.

## 2 The Role of Learning

As Cogley and Sargent (2008) argue, if we assume that agents know the underlying parameters of our model with certainty, as is the standard assumption im-

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<sup>6</sup>Here we make an important distinction between a default rate and the incidence of default. A default rate is the fraction of households who find it optimal to default on a specific debt contract at a given date. The incidence of default, on the other hand, is the fraction of households who find it optimal to default across all available debt contracts at a given date. It is the incidence of default in our model that corresponds to the bankruptcy filing rate that we observe in the data.

posed by rational expectations, we are implicitly assuming that all learning has been completed. Although this assumption may be innocuous and serve as a convenient simplification in many cases, there is substantial evidence that the post-1984 period, commonly referred to as the Great Moderation, was fundamentally different than the period that preceded it (see, for example, the work of Kim and Nelson (1999), Benati and Surico (2009) and McConnell and Perez-Quiros (2000), among others). Some authors (Lubik and Schorfheide (2004) and Clarida, Gali, and Gertler (2000)) argue that the significant reduction in the volatility of macroeconomic time series during the post-1984 period was due to improved economic policy. Others (Stock and Watson (2003), Sims and Zha (2006) and Gambetti, Pappa, and Canova (2008)) have argued that the Great Moderation was simply a sequence of favorable aggregate shocks (good luck) and that there was actually no change in the underlying data generating process.

While we are agnostic about the causes of the Great Moderation, we recognize that imposing rational expectations would require us to assume the agents in our model knew with certainty whether or not the underlying data generating process had changed in 1984, not to mention the precise values of any new or updated model parameters. We see this as an unreasonably stark assumption, especially in light of the fact that the first papers to document a reduction in volatility during this period did not appear until the late 1990's. Given that it took quite a long time even for academic economists to begin to question whether or not the economic environment had fundamentally changed, and furthermore that a lively debate continues in the literature, we cannot reasonably expect the average agent in our model to know with certainty in 1984 one way or the other. We view learning as the obvious response to this critique. It takes time for economic agents to change their beliefs, and we argue that we are better able to understand the behavior of households and creditors during this period by explicitly modeling their dynamic learning process.

There is also evidence that accounting for the dynamic learning process of creditors is consistent with actual credit card industry practices during this period. Throughout the 1980's and 1990's it was common for lenders to use linear models to evaluate the credit-worthiness of potential borrowers. In addition, lenders tended to update the parameters of their models frequently – not less than once every two years – by re-running their regressions using the most up-to-date data on consumer default decisions. Driven mainly by concerns about the possibility of population drift, creditors' behavior suggests that the weight they attributed to historical data in forming their beliefs about current consumer default probabilities was rather low.<sup>7</sup> Not only

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<sup>7</sup>Population drift refers to the potential for the distribution of the characteristics of a population to change over time. See Thomas (2000) for a detailed discussion of the history of credit scoring

does this observation support our view that learning was an important factor during this period, it also points us toward the appropriate type of learning to consider. While the theoretical model that we develop here is general enough to encompass a variety of learning algorithms, we focus on the case of constant gain learning for our quantitative exercises. In addition to the fact that our approach is consistent with actual creditor behavior during this period, constant gain learning is also preferred to recursive least squares when agents suspect that the economy may be undergoing a period of structural change.<sup>8</sup>

It is well known that learning in models of this type tends to be self-referential in nature, meaning that the beliefs of agents directly influence market outcomes, which in turn affect agents' expectations about the future. The model we develop here is self-referential in the following sense: When the economy experiences a sequence of favorable aggregate shocks, the realized default rate for any given loan contract is less than anticipated. In response, creditors reduce their expectations, leading to lower equilibrium interest rates. Lower interest rates allow households to more easily roll over their debt, which further reduces the realized default rate for any given loan contract.<sup>9</sup> The fact that creditors in our model learn about future default probabilities imparts momentum into their expectations and amplifies the economy's response to exogenous shocks.

As has become standard in the literature on statistical learning, we adopt the model of anticipated utility originally developed by Kreps (1998) and first used in applied work by Sargent (1999) to analyze trends in U.S. inflation. In this framework, agents reoptimize at each point in time given their current beliefs. Cogley and Sargent (2008) demonstrate that when agents are not too risk averse, anticipated utility models closely approximate the results that are generated by models in which agents also learn, but are considered to be fully rational in the Bayesian sense.<sup>10</sup> Moreover, anticipated utility models have the advantage of being significantly easier to implement than models that use Bayesian learning rules. To our knowledge, this paper is the first to use anticipated utility in a model of optimal default.<sup>11</sup>

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practices used by lenders to decide whether or not to grant credit to those who apply.

<sup>8</sup>See, for example, Adam, Marcet, and Nicolini (2008).

<sup>9</sup>If this was the only effect, our model would not be able to produce a simultaneous rise in debt and default. A rise in the incidence of default occurs in our model because households endogenously take on more debt.

<sup>10</sup>More precisely, when agents have constant relative risk aversion preferences with a coefficient of 2 or less, the predictions of a model in which agents use recursive least squares are nearly identical to those generated by a model in which agents use Bayesian learning.

<sup>11</sup>This paper is also the first, to our knowledge, to allow for aggregate uncertainty in a model of optimal default. We demonstrate that learning in an environment with aggregate uncertainty is

Statistical learning algorithms, such as constant gain learning, have recently been considered in many different contexts and have been shown to improve the quantitative performance of existing models.<sup>12</sup> Carceles-Poveda and Giannitsarou (2008) and Adam et al. (2008), for example, use learning of this kind to shed new light on a variety of asset pricing puzzles. Eusepi and Preston (2008) introduce constant gain learning into a standard real business cycle model and find that it generates increased volatility in hours worked, thereby bringing the model’s predictions closer to the data. Our paper contributes to these findings by illustrating that learning is a quantitatively important factor in explaining the rise of consumer debt-to-income ratios and bankruptcy filing rates from 1984 to 1998.

### 3 Model

We consider a defaultable debt model in the spirit of Eaton and Gersovitz (1981). Time is discrete and infinite. The economy is populated by a measure one of infinitely lived households that receive a stochastic endowment  $y_t$  each period. The process from which this endowment is drawn depends on the realization of an aggregate state variable  $s_t \in S = \{s^1, \dots, s^N\}$ , where  $S$  is a time-invariant set and  $s_t$  evolves according to a Markov process with transition matrix  $\Pi$ . The only asset is a one-period, unsecured and unconditional discount bond which trades at a price set by a pool of risk neutral, perfectly competitive creditors.

Each period households choose whether or not to repay their debt. A defaulting household enters bankruptcy, which we model after Chapter 7 of the U.S. bankruptcy code.<sup>13</sup> A household that defaults in our model is relieved of their outstanding debt

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the key feature that allows our model to closely match the data.

<sup>12</sup>Evans and Honkapohja (2001) provide a comprehensive overview of the literature on statistical learning, its theoretical properties, and potential applications.

<sup>13</sup>The U.S. bankruptcy code offers consumers two choices when filing for bankruptcy protection: Chapter 7 and Chapter 13. A household that chooses to file under Chapter 7 is relieved of all outstanding debt obligations in exchange for their assets net of any personal exemptions. A household that chooses to file under Chapter 13, on the other hand, agrees to pay back a portion of their outstanding debt obligations over a 3-5 year period in exchange for the ability to keep their assets. In either case, the household is not allowed to refile under the same chapter for a period of 6 years, and a record of their bankruptcy is maintained on their credit report for a period of 10 years. The conditions of default in our model are chosen to match Chapter 7 of the U.S. bankruptcy code, which accounts for approximately 70% of bankruptcy filings over the period under consideration. Moreover, given the choice between Chapters 7 and 13, a household would only choose Chapter 13 if they have assets that they would like to keep but would otherwise lose by filing under Chapter 7. Since there is only one asset in our model, a defaulting household will inevitably have a negative asset position, and therefore will always prefer to file under Chapter 7.



obligations and is punished with an endowment cost and restricted access to credit markets in future periods. In the period of default, the household is prohibited from interacting in credit markets. In the period following default, the household's credit report is marked with a bankruptcy flag. Households with a bankruptcy flag are considered to be in a state of bad credit standing which persists for a random number of periods. While in bad credit standing, a household does not incur any additional costs and may save, but is restricted from borrowing.

### 3.1 Timing of Events

In any period  $t$ , the timing of events taking place in the model is as follows:

1. Households and creditors enter with prior beliefs about the transition matrix and default probabilities, respectively.
2. The aggregate state  $s_t$  and idiosyncratic endowments  $y_t$  are realized.
3. Given their prior beliefs and the current aggregate state, households form their posterior beliefs about the aggregate state transition matrix.
4. Creditors announce a bond price schedule consistent with their prior beliefs about household default probabilities.
5. Households who are in bad credit standing have their bankruptcy flag removed and regain full access to credit markets with probability  $\theta$ .
6. Given their posterior beliefs and bond prices, households in good credit standing make default, consumption, and borrowing decisions, while households in bad credit standing make consumption and saving decisions.
7. Given their prior beliefs and observed default decisions, creditors form their posterior beliefs about household default probabilities.

### 3.2 Household's Problem

Each period households receive a stochastic endowment  $y$ , the log of which evolves according to the following first-order autoregressive process:

$$\log(y_t) = (1 - \rho)\mu_{s_t} + \rho \log(y_{t-1}) + \varepsilon_t \tag{1}$$

where  $\varepsilon_t \sim N(0, \eta_{s_t}^2)$ . The unconditional mean of the endowment process and the variance of the idiosyncratic endowment shock depend on the realization of the aggregate state  $s_t$ .

A household in good credit standing ( $G$ ) observes the bond price schedule set by creditors and chooses whether to default ( $D$ ) or repay their debt obligations ( $R$ ):

$$V_t^G(b, y; s) \equiv \max_{R, D} \{V_t^R(b, y; s), V_t^D(y; s)\}, \quad (2)$$

where  $V_t^D(y; s)$  represents the value of defaulting and  $V_t^R(b, y; s)$  is the value associated with repaying their debt at date  $t$ .<sup>14</sup> We adopt the convention that  $b > 0$  represents a household with positive assets, while  $b < 0$  represents a household with negative assets, or positive debt. Note that since a defaulting household is relieved of their debt obligations, the value of defaulting is independent of  $b$ .

If the household repays its debt, it then optimally chooses consumption and its asset position with which it leaves the period. The value of this option is given by:

$$V_t^R(b, y; s) = \max_{b'} u(c) + \beta \mathbb{E}_t [V_t^G(b', y'; s') | y; s] \quad (3)$$

subject to

$$c + q_t(b', y; s)b' = y + b$$

where  $\mathbb{E}_t$  are household expectations conditional on their beliefs about the aggregate state transition matrix at date  $t$ . A household that does not default remains in good credit standing and faces the same problem in the following period of whether or not to default and thus receives an expected continuation value of  $\mathbb{E}_t [V_t^G(b', y'; s') | y; s]$ .

If the household chooses to default, they are relieved of their outstanding debt obligations in exchange for an endowment cost. We assume that this endowment cost is weakly increasing in the household's endowment in order to discourage high income households from choosing to default.<sup>15</sup> The household is also prohibited from

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<sup>14</sup>We adopt the anticipated utility model developed by Kreps (1998) and used by Sargent (1999), among others. In this framework, households reoptimize at each point in time given their current beliefs. For this reason, household decision rules and value functions, which depend on the household's beliefs about the aggregate state transition matrix at date  $t$ , are time-dependent and thus are appropriately labeled with time subscripts.

<sup>15</sup>We think this is an important feature of our model since otherwise a household with high income and a large amount of outstanding debt would have an incentive to game the system by filing for bankruptcy. The form of endowment cost that we consider is similar to that imposed by Arellano (2008) in a model of sovereign default. A key difference is that in our model a household only incurs this cost in the period of default, whereas in Arellano (2008) the sovereign must pay this cost until they are allowed to reenter international credit markets.

saving in the period in which they default. Hence, a defaulting household simply consumes their endowment net of any bankruptcy costs. The value of defaulting in the current period is thus given by:

$$V_t^D(y; s) = u(c) + \beta \mathbb{E}_t [V_t^B(0, y'; s') | y; s] \quad (4)$$

where

$$c = \min \{y, \psi \mathbb{E}[y | s]\}$$

and  $V_t^B(0, y'; s')$  is the value of a household that has a bankruptcy flag on their credit report and so is considered to be in bad credit standing.

Households in bad credit standing are restricted from borrowing.<sup>16</sup> But since the U.S. bankruptcy code does not prohibit asset accumulation after the discharge of debt, we allow households to save. Each period following default, the household has their bankruptcy flag removed and regains full access to credit markets with probability  $\theta$ , while with probability  $1 - \theta$  the bankruptcy flag remains on their credit report. The value of a household in this post-default state is given by:

$$V_t^B(b, y; s) = \max_{b' \geq 0} u(c) + \beta \mathbb{E}_t [\theta V_t^G(b', y'; s') + (1 - \theta) V_t^B(b', y'; s') | y, s] \quad (5)$$

subject to

$$c + q_t(b', y; s)b' = y + b.$$

### 3.3 Bond Prices

The bond price schedule is determined in equilibrium by the profit maximizing behavior of a pool of perfectly competitive, risk neutral creditors that can borrow and lend in international markets at the exogenously given, risk free rate  $r$ . Creditors in our model face a proportional transaction cost  $\tau > 0$  of making loans to households. One should think of  $\tau$  as representing the cost to a lender of verifying a household's income prior to issuing a loan. The assumptions of risk neutrality and perfect competition imply that creditors must earn zero expected profits on each credit contract

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<sup>16</sup>Musto (2004) argues that creditors view default as an adverse signal about a household's future ability to repay their debt. Consequently, access to credit for households that have a bankruptcy filing on their credit report may be available on prohibitively tough terms or may not be available at all. Musto finds that this effect tends to last until the household's credit report is cleared of their bankruptcy flag which occurs by law 10 years after the date at which their debt was discharged.

they enter into with a household. Furthermore, the ability of creditors to price loans based on the loan size, the household's income, and the aggregate state rules out cross-subsidization. As a result, bond prices must fully reflect the expected default probability for a loan with these given characteristics. Hence, the bond price for a contract where  $b' < 0$  is given by:

$$q_t(b', y; s) = \frac{1 - \mathbb{E}_t [D_t(b', y'; s') | y; s]}{(1+r)(1+\tau)}, \quad (6)$$

where  $D_t(b, y; s)$  is an indicator function taking the value of one if the household defaults and zero otherwise:

$$D_t(b, y; s) = \left\{ \begin{array}{ll} 1 & \text{if } V_t^D(y; s) > V_t^R(b, y; s) \\ 0 & \text{otherwise} \end{array} \right\}. \quad (7)$$

Since households that save will never find it optimal to default, they carry no default risk. Moreover, we assume there are no transaction costs ( $\tau = 0$ ) associated with accepting deposits since income verification for the case in which households want to save is unnecessary. Thus, the bond price for a contract where  $b' > 0$  is equal to  $1/(1+r)$ .

### 3.4 Information and Learning

Households and creditors have disjoint and incomplete information about the underlying model parameters. Each must use the information that they have available to form expectations about the future in order to be able to act optimally. On one hand, households must learn about the aggregate state transition matrix so that they can make optimal consumption and savings decisions. Creditors, on the other hand, must learn about household default probabilities in order to price household debt appropriately. We assume that both households and creditors use linear, statistical learning rules in order to form their posterior beliefs given their prior and the realization of relevant economic variables. The details of these dynamic learning algorithms are outlined below.

#### 3.4.1 Households

Households know the parameters governing their idiosyncratic and state-dependent endowment process but are uncertain about the transition probabilities governing the aggregate state. Given an initial prior  $\Pi_0$ , households learn over time about the aggregate state transition matrix by observing the realized sequence of aggregate states and using a linear updating rule to form their posterior beliefs.

Suppose that the observed transition at date  $t$  is from aggregate state  $s^i$  to  $s^j$ , and let  $\Pi_t^k$  denote the  $k^{th}$  row of  $\Pi_t$ . If  $\Pi_{t-1}$  is a household's prior belief about the aggregate state transition matrix at date  $t$ , then their posterior beliefs given the realized transition are:

$$\Pi_t^k = \left\{ \begin{array}{ll} \gamma_{h,t}1^j + (1 - \gamma_{h,t})\Pi_{t-1}^k & \text{if } k = i \\ \Pi_{t-1}^k & \text{otherwise} \end{array} \right\} \quad (8)$$

where  $1^j$  is a row vector with a 1 as the  $j^{th}$  element and 0's elsewhere and  $\gamma_{h,t}$  is the gain parameter which governs the relative weight given to a household's prior when forming posterior expectations. Since the household receives no additional information about transitions for states  $s^k$  with  $k \neq i$ , those rows of the transition matrix are not updated and remain equal to the household's prior beliefs.

### 3.4.2 Creditors

Creditors observe the aggregate state as well as the endowment of any household with whom they make a debt contract but do not observe the household's endowment in the following period nor do they know the parameters of their endowment process. While creditors can condition their loan contracts on all of the relevant state variables for the household, these assumptions imply that they are unable to compute the expected default probability for any given loan type. For this reason, creditors must form beliefs about household default probabilities by observing the sequence of realized default rates for each loan type.

Let  $\mathbb{E}_t [D_t(b', y'; s')|y; s]$  be the creditors' current expected default probability of a household who borrows  $b'$  with endowment  $y$  in state  $s$ . Creditors update their beliefs using the new information they obtain by observing actual default rates in the economy each period. When observed default rates differ from their expectations, creditors use their forecast errors to update their beliefs. Let  $DR_t(b'; s'|y)$  represent the observed default rate at date  $t$  given state  $s'$  for households that borrowed an amount  $b'$  with endowment  $y$  at date  $t-1$ . Creditors' posterior beliefs are then given by:

$$\mathbb{E}_{t+1} [D(b', y'; s')|y; s] = \gamma_{c,t}DR_t(b'; s'|y) + (1 - \gamma_{c,t})\mathbb{E}_t [D(b', y'; s')|y; s]. \quad (9)$$

The parameter  $\gamma_{c,t}$  governs the weight creditors place on new information relative to their prior when updating their beliefs. Given that creditors must announce a bond price schedule prior to household default decisions,  $DR_t(b'; s'|y)$  is the most recent

default information available to creditors when setting the bond price schedule at date  $t + 1$ .

The fact that creditors learn about an endogenous object, instead of an exogenous process such as the aggregate state, gives our model a self-referential property that operates in the following way: When a favorable (adverse) aggregate shock occurs, default rates are below (above) creditor expectations. Given their updating rule, expectations about default probabilities are revised downward (upward). In the following period, lower (higher) interest rates make it easier (harder) for a household to roll over its debt, thus leading to even lower (higher) default rates than expected. This mechanism imparts momentum into creditor beliefs, amplifying the model's response to a sequence of favorable aggregate shocks, such as that realized during the Great Moderation.

### 3.5 Equilibrium

***Definition** An equilibrium for this economy is sequences of household decision rules and beliefs  $b'_t(b, y; s)$ ,  $D_t(b, y; s)$ , and  $\Pi_t$ , and creditor beliefs  $\mathbb{E}_t [D_t(b', y'; s')|y; s]$ , such that, given initial beliefs for households and creditors  $\Pi_0$  and  $\mathbb{E}_0 [D_0(b', y'; s')|y; s]$ , an initial distribution of households over bonds and endowments  $\Phi_0$ , learning rules, and sequences of bond prices  $q_t(b', y; s)$ , aggregate states  $s_t$ , and endowment shocks  $y_t^i$ , the decision rules solve each household's problem at every date  $t$  and bond prices maximize creditors' profits at every date  $t$ .*

## 4 Analytical Results

In this section we first consider a simplified version of our full model from which we can derive results that provide intuition for our quantitative results that follow. We also prove a few general results for a more fully articulated setting. To focus on the effect of creditor learning, in this section we assume that households do not learn about the transition matrix in response to the observed sequence of aggregate shocks. In addition, we assume in what follows that a defaulting household is forever restricted from borrowing in order to maintain analytical tractability. That is, we assume that  $\gamma_h = 0$  and  $\theta = 0$  in this section.

### 4.1 An Illustrative Example

Several of the main results of our paper can be shown analytically using a simplified version of our full model. Prior to introducing the example, we establish several

intermediate results that will be useful for this section and the next.

#### 4.1.1 Preliminary Results

Let  $\tilde{b}_t^{y,s}$  be the value of debt that makes a household with endowment  $y$  in state  $s$  indifferent between repaying its debt obligations and defaulting, so that  $V_t^D(y; s) = V_t^R(\tilde{b}_t^{y,s}, y; s)$ . Our first result establishes that households with debt greater than  $\tilde{b}_t^{y,s}$  find it optimal to default.

**Theorem 4.1** *A household with endowment  $y$  in state  $s$  finds it optimal to default if they have debt obligations  $b < \tilde{b}_t^{y,s}$ .*

**Proof** Notice that  $V_t^D(y; s)$  is independent of  $b$  while  $V_t^R(b, y; s)$  is increasing in  $b$ . Consider some  $b < \tilde{b}_t^{y,s}$ . Then  $V_t^R(b, y; s) < V_t^R(\tilde{b}_t^{y,s}, y; s) = V_t^D(y; s)$ . Hence, it is optimal for the household to default on their debt.

Our next result demonstrates that the default thresholds  $\tilde{b}_t^{y,s}$  are decreasing in the bond price schedule. First, we prove an intermediate result that our problem is a contraction.

**Lemma 4.2** *Define the operator  $T_q$  as follows:*

$$(T_q V_t^G)(b, y; s) = \max \left\{ V_t^D(y; s), \max_{b'} \left\{ u(y + b - q_t(b', y; s)b') + \beta \mathbb{E} [V_t^G(b', y'; s')] \right\} \right\}.$$

*Then,  $T_q$  is a contraction mapping, so that there exists a unique fixed point of this mapping denoted  $V_{t,q}^*(b, y; s)$ .*

**Proof** See Appendix A for proof.

**Theorem 4.3** *The default thresholds  $\tilde{b}_t^{y,s}$  decrease in response to an increase in the bond price schedule such that for all  $\{b, y, s\}$ ,  $q_{t+1}(b, y; s) \geq q_t(b, y, s)$ .*

**Proof** See Appendix A for proof.

The intuition for this result is that as bond prices rise, interest rates fall by definition, and hence any household must be at least as well off.<sup>17</sup> Since defaulting households are restricted from borrowing, the value of defaulting is independent of changes in the bond price schedule. This implies that the value of repaying debt is increasing in the bond price schedule, and therefore default thresholds must fall as a result of an increase in bond prices.

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<sup>17</sup>Recall that the interest rate paid on household savings is always equal to the risk free rate since a household with  $b \geq 0$  will never find it optimal to default.

### 4.1.2 Example

The key mechanism that allows our model to generate an endogenous increase in debt and bankruptcies is the way in which bond prices adjust in response to a sequence of positive aggregate shocks – shocks with relatively low default rates. In order to illustrate how this channel operates, consider the case in which  $S \equiv \{c, e\}$ , where ‘c’ represents a contraction and ‘e’ represents an expansion. Let

$$\Pi = \begin{bmatrix} \pi_{ee} & \pi_{ec} \\ \pi_{ce} & \pi_{cc} \end{bmatrix},$$

where  $\pi_{ij} = \Pr\{s_{t+1} = j | s_t = i\}$  and we assume  $\pi_{ii} > 1/2$ . In addition, let each household’s idiosyncratic endowment take a value  $y \in \{y_L, y_H\}$ ,  $y_L < y_H$ , where

$$P^i = \begin{bmatrix} p_{HH}^i & p_{HL}^i \\ p_{LH}^i & p_{LL}^i \end{bmatrix},$$

and  $p_{mj}^i = \Pr\{y' = j | y = m; s_{t+1} = i\}$  and we assume  $p_{jj}^i > 1/2$ . We also assume that the persistence of  $y_H$  and  $y_L$  are lower and higher, respectively, in contraction,  $p_{HH}^e \geq p_{HH}^c$  and  $p_{LL}^e \leq p_{LL}^c$ .<sup>18</sup>

To illustrate the effect of learning in our model, suppose that creditors begin at date 0 with beliefs that are consistent with the transition matrices of the true data generating process,  $\Pi$  and  $P^i$ . Let  $\bar{b}_t \equiv \max_{y,s} \{\tilde{b}_t^{y,s}\}$  and  $\underline{b}_t \equiv \min_{y,s} \{\tilde{b}_t^{y,s}\}$ . Then from the perspective of a creditor, lending a household  $b' \in [\bar{b}_t, 0)$  is risk free since the household will repay their debt in the following period with probability one. Hence, the corresponding bond prices are given by

$$q_0(b', y; s | b' \in [\bar{b}_t, 0)) = \frac{1}{(1+r)(1+\tau)}.$$

On the other hand, no creditor would ever lend a household  $b' < \underline{b}_t$  since the household will default with probability one in the following period. Hence, the corresponding bond prices are given by

$$q_0(b', y; s | b' < \underline{b}_t) = 0.$$

We now consider a debt contract between a household and creditor with a non-trivial default probability. In particular, suppose that  $\tilde{b}_0^{L,c} = \max_{y,s} \{\tilde{b}_0^{y,s}\}$ <sup>19</sup> and let  $\hat{b}_t \equiv \max\{\tilde{b}_t^{H,e}, \tilde{b}_t^{L,e}, \tilde{b}_t^{H,c}\}$ . Then lending a household  $b' \in [\hat{b}_t, \tilde{b}_t^{L,c})$  is risky since

<sup>18</sup>In this sense, a contraction is riskier than an expansion.

<sup>19</sup>This condition is satisfied in our calibrated model.



if the economy transitions to state  $c$  and the household receives endowment  $y_L$  in the following period, they will default. At date 0 creditors' expected probability of default is

$$\mathbb{E}_0 \left[ D(b', y'; s') | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}], y; s \right] = \pi_{sc} p_{yL}^c,$$

and the corresponding bond price is given by

$$q_0 \left( b', y; s | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}] \right) = \frac{1 - \pi_{sc} p_{yL}^c}{(1+r)(1+\tau)}.$$

To see what happens when creditors learn, consider the evolution of creditors' expected default probability for a loan of size  $b' \in [\hat{b}_0, \tilde{b}_0^{L,c}]$  made to a household with endowment  $y_H$  in state  $e$  in response to a sequence of states  $s_t = e$  for all  $t \geq 0$ . Conditional on  $s_0 = e$ , the household honors its debt obligation regardless of whether they receive  $y_L$  or  $y_H$  since  $b' > \max\{\tilde{b}_0^{H,e}, \tilde{b}_0^{L,e}\}$ . Hence, the realized default rate  $DR_0(b'; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}], y_H) = 0$ . Creditors observe this default rate and update their beliefs according to (9):

$$\begin{aligned} \mathbb{E}_1 \left[ D(b', y'; s') | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}], y_H; e \right] &= \gamma_c DR_0(b'; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}], y_H) \\ &+ (1 - \gamma_c) \mathbb{E}_0 \left[ D(b', y'; s') | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}], y_H; e \right], \end{aligned}$$

or

$$\mathbb{E}_1 \left[ D(b', y'; s') | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}], y_H; e \right] = (1 - \gamma_c) \pi_{ec} p_{HL}^c.$$

The corresponding bond price is then given by

$$q_1 \left( b', y_H; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}] \right) = \frac{1 - (1 - \gamma_c) \pi_{ec} p_{HL}^c}{(1+r)(1+\tau)}.$$

Given that  $\gamma_c \in (0, 1)$ ,

$$\mathbb{E}_1 \left[ D(b', y'; s') | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}], y_H; e \right] < \mathbb{E}_0 \left[ D(b', y'; s') | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}], y_H; e \right]$$

and

$$q_1 \left( b', y_H; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}] \right) > q_0 \left( b', y_H; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}] \right).$$

Recall Theorem 4.3 which states that  $\tilde{b}_t^{y,s}$  is decreasing in  $q$ . Hence, at date  $t \geq 1$ , either  $b' \in [\hat{b}_t, \tilde{b}_t^{L,c})$ , in which case the loan is risky, or  $b' \geq \tilde{b}_t^{L,c}$ , in which case the loan is risk free. Yet in either case, as long as the economy remains in the expansion state, we have that  $DR_t(b'; e|b' \in [\hat{b}_0, \tilde{b}_0^{L,c}), y_H) = 0$ . Iterating on (9), it follows that

$$\mathbb{E}_t \left[ D(b', y'; s') | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}), y_H; e \right] = (1 - \gamma_c)^t \pi_{ec} p_{HL}^c.$$

The corresponding bond price is then given by

$$q_t \left( b', y_H; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}) \right) = \frac{1 - (1 - \gamma_c)^t \pi_{ec} p_{HL}^c}{(1 + r)(1 + \tau)}.$$

It follows that

$$\lim_{t \rightarrow \infty} \mathbb{E}_t \left[ D(b', y'; s') | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}), y_H; e \right] = 0,$$

while

$$\lim_{t \rightarrow \infty} q_t \left( b', y_H; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}) \right) = \frac{1}{(1 + r)(1 + \tau)}.$$

In the limit, for any  $\gamma_c \in (0, 1)$ , creditors completely discount the probability of transitioning from expansion to contraction and bond prices adjust accordingly. This leads creditors to reduce the default premium applied to the bond to zero, even though a household will default on the contract if the economy transitions to contraction and the household receives the low endowment shock in the following period. Note that the rate at which bond prices converge to the risk free lending rate is governed by the choice of  $\gamma_c$ .

## 4.2 Theoretical Results

In this section we prove the two main theoretical results of our paper: (1) If creditors use this learning algorithm to update their beliefs about the probability of default, bond prices rise and household debt increases in response to a sequence of favorable aggregate shocks, and (2) if the economy realizes a sequence of favorable aggregate shocks forever, then creditors' expectations and the bond price schedule will converge with probability one.<sup>20</sup>

Define the realized default rate at date  $t$  for households with debt  $b'$  in state  $s'$  that had endowment  $y$  in the previous period as

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<sup>20</sup>We will formally define what we mean by a 'favorable' aggregate shock in what follows.

$$DR_t(b'; s' | y) \equiv \sum_{y' \in Y} \mathbb{I}(b' < \tilde{b}_t^{y', s'}) \Pr[y' | y; s'].$$

where  $\mathbb{I}(\cdot)$  is the indicator function taking a value of 1 if the interior argument is true and 0 otherwise. The following theorem states that, given the learning algorithm previously detailed, the realization of aggregate states for which the actual default rate is less (greater) than expected results in higher (lower) bond prices. This result is a direct application of the learning algorithm used by creditors in our model.

**Theorem 4.4** *Let  $X_t(b', y) \equiv \{s' \in S : DR_t(b'; s' | y) \leq \mathbb{E}_t [D(b', y'; s') | y; s]\}$ . If  $s_t \in X_t(b', y)$ , then  $\mathbb{E}_{t+1} [D(b', y'; s') | y; s] \leq \mathbb{E}_t [D(b', y'; s') | y; s]$  and  $q_{t+1}(b, y; s) \geq q_t(b, y; s)$ . Otherwise,  $\mathbb{E}_{t+1} [D(b', y'; s') | y; s] \geq \mathbb{E}_t [D(b', y'; s') | y; s]$  and  $q_{t+1}(b, y; s) \leq q_t(b, y; s)$ .*

**Proof** See Appendix A for the proof.

The following corollary presents the first of our two main theoretical results. It states that observing states for which the realized default rate is less than that expected by creditors leads to a rise in household borrowing on the extensive and intensive margins.

**Corollary 4.5** *Suppose  $s_{t-1} = s_t = s_{t+1} \equiv \hat{s}$  where  $\hat{s} \in X_t(b', y)$  for all  $(b', y)$ . Let  $i$  index households. Then*

$$\int q_{t+1} b_{t+1}(i) di \leq \int q_t b_t(i) di,$$

and

$$\int \mathbb{I}(b_{t+1}(i) < 0) di \geq \int \mathbb{I}(b_t(i) < 0) di.$$

**Proof** Following Theorem 4.4, the bond price schedule increases in response to this sequence of shocks. Thus, the relative cost of borrowing (consumption today) declines. As a result, the income and substitution effects cause an increase in borrowing such that  $q_{t+1}(b_{t+1}, y; \hat{s}) b_{t+1}(b, y; \hat{s}) \leq q_t(b_t, y; \hat{s}) b_t(b, y; \hat{s})$  for all  $b$  and  $y$  and each household. The second result is implied by this fact.

We now establish what happens in the limit as the economy realizes an infinite sequence of aggregate shocks for which the actual default rate is less than the expected default rate at every date  $t$ .

**Theorem 4.6** *Suppose  $s_t \in X_t(b', y)$  for all  $t \geq 0$ . Then:*

1.  $\lim_{t \rightarrow \infty} \mathbb{E}_t [D(b', y'; s') | y; s] = \tilde{\mathbb{E}} [D(b', y'; s') | y; s] \in [0, \mathbb{E}_0 [D(b', y'; s') | y; s]]$
2.  $\lim_{t \rightarrow \infty} q_t(b, y; s) = \tilde{q}(b, y; s) \in [q_0(b, y; s), 1 / ((1 + r)(1 + \tau))]$ .

**Proof** See Appendix A for the proof.

This result tells us that if the economy repeatedly experiences favorable aggregate shocks that produce default rates below the current period expectations, then creditor expectations and bond prices must converge in the limit. Moreover, we know that creditor expectations are bounded above by their initial expectations, and bond prices are no less than their initial value.

## 5 Explaining the Rise in Consumer Debt and Bankruptcies

In this section we describe how we calibrate our model and the experiment that we conduct to test how much of the simultaneous rise in consumer debt and bankruptcies between 1984 and 1998 can be accounted for by learning.

### 5.1 Calibration

We define a period in our model to be one year. In order to simulate our economy we must first parameterize the process for the aggregate state. To do so we discretize the aggregate state into four values by classifying the years 1890 through 1998 based on the unemployment rate (low or high) and NBER recession dates (expansion or contraction).<sup>21</sup> Using data from the Panel Study of Income Dynamics (PSID), Storesletten et al. (2004) establish that income dispersion increases substantially during recessions relative to expansions. Conducting a similar analysis to those authors, we find that mean income tends to be high (low) when the unemployment rate is low (high). The set of aggregate states  $S$  thus contains four elements: expansion (e) or contraction (c), combined with either high (h) or low (l) mean income. Hence,  $S = \{(e, h), (c, h), (c, l), (e, l)\}$ . We construct the following transition matrix for the aggregate state by counting the transitions observed between 1890 and 1983 implied by our classification of years:

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<sup>21</sup>Our unemployment rate series is constructed using data from Romer (1986) for the years 1890 to 1930, Lebergott (1964) for 1931 to 1940, and the Bureau of Labor Statistics from 1941 onward.

$$\Pi = \begin{bmatrix} 0.55 & 0.35 & 0.10 & 0.00 \\ 0.31 & 0.35 & 0.27 & 0.08 \\ 0.20 & 0.00 & 0.27 & 0.53 \\ 0.67 & 0.16 & 0.00 & 0.17 \end{bmatrix},$$

where  $[\Pi]_{ij}$  represents the probability of transitioning from state  $i$  to state  $j$ .

We take the persistence and state-dependent standard deviation of the household's income process directly from Storesletten et al. (2004). In particular, we use  $\rho = 0.941$ ,  $\eta_e = 0.088$  and  $\eta_c = 0.162$ . Hence, idiosyncratic income shocks are roughly twice as volatile during contractions than expansions. Our own estimates for the state-dependent mean of the income process are  $\mu_h = 7.95$  and  $\mu_l = 7.89$ .<sup>22</sup> Given that these values are in logs, we can conclude that the mean of the income process when the unemployment rate is low is roughly 6% higher than when the unemployment rate is high. Finally, we discretize the endowment process for each of the four aggregate states using the method employed by Tauchen and Hussey (1991).

The remaining parameters are chosen as follows. We assume households have CRRA preferences and set  $\sigma = 2$ , as is standard in the literature. Following Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) we set  $\beta = 0.91$ .<sup>23</sup> The risk free rate is taken as the average real return on 1-year U.S. Treasury bills between 1984 and 1998. In order to match an average exclusion from credit markets of six years, we set  $\theta = 0.2$ . This implies that households in our model are, on average, able to refile for bankruptcy after six years which is consistent with the U.S. bankruptcy code.

Households and creditors are assumed to use constant gain learning, and therefore assign a lower weight to past observations in order to protect themselves against the possibility of structural change. We think that this form of learning is appropriate since it closely resembles how creditors actually behaved during this period, results in more accurate forecasts than recursive least squares when agents are concerned about the potential for structural change, and is more tractable than Bayesian learning while producing strikingly similar results.<sup>242526</sup> In order to implement this dynamic learning algorithm, we must specify the gain used by households and creditors

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<sup>22</sup>See Appendix B for a detailed description of our estimation procedure.

<sup>23</sup>Similarly low discount factors are often needed to achieve empirically accurate default rates in models of consumer and sovereign default (see Aguiar and Gopinath (2006) for sovereign default).

<sup>24</sup>See Thomas (2000) for a discussion of credit industry practices during the 1980's and 1990's.

<sup>25</sup>Adam et al. (2008) make a convincing case for the use of constant gain learning rather than recursive least squares in the context of an asset pricing model.

<sup>26</sup>Cogley and Sargent (2008) demonstrate that the results generated under statistical and Bayesian learning are nearly indistinguishable when agents are not too risk averse.

when updating their beliefs. Following Adam et al. (2008) and Eusepi and Preston (2008), we consider only a modest deviation from rational expectations on the part of households and set  $\gamma_h = 0.05$ . Since creditors updated their models at least once every two years and discarded past observations in order to protect themselves against population drift, we set  $\gamma_c = 0.50$  which implies that creditors place equal weight on their prior and the realized default rate when forming their posterior beliefs in any given period.

Finally, we use simulated method of moments to pin down the values of  $\psi$  and  $\tau$  that allow our model to most closely match the average unsecured debt-to-income ratio and consumer bankruptcy filing rate in 1983.<sup>27</sup> Our baseline parameterization is summarized in Table 1.

Table 1: Calibrated Values

Parameter	Value	Source / Target
$\sigma$	2	standard
$\beta$	0.91	Chatterjee et al. (2007)
$r$	0.017	real return on 1 yr US T-bills (1983-1998)
$\theta$	0.2	avg exclusion from credit markets of 6 yrs
$\rho$	0.941	Storesletten et al. (2004)
$\eta_e$	0.088	Storesletten et al. (2004)
$\eta_c$	0.162	Storesletten et al. (2004)
$\mu_h$	7.95	own estimate using PSID data
$\mu_l$	7.89	own estimate using PSID data
$\gamma_h$	0.05	own estimate
$\gamma_c$	0.50	Thomas (2000)
$\psi$	0.5	bankruptcy filing rate in 1983 of 0.2%
$\tau$	0.040	debt-to-income ratio in 1983 of 4.9%

## 5.2 Quantitative Results

To analyze the ability of our model with learning to account for the rise in consumer debt and bankruptcy filing rates over this period, we conduct the following experiment:

<sup>27</sup>This consists of repeatedly simulating the model up to 1983 given the observed sequence of aggregate shocks and the assumption that  $\gamma_h = \gamma_c = 0$  for a finite grid of  $\psi$  and  $\tau$ .

1. Set  $\mathbb{E}_0 [D(b', y'; s') | y; s]$  and  $\Pi_0$  to be consistent with the true data generating process up to 1983.
2. Simulate the economy without household or creditor learning ( $\gamma_h = \gamma_c = 0$ ) given the observed sequence of aggregate shocks from 1890 to 1983.<sup>28</sup>
3. Then simulate the economy with household and creditor learning ( $\gamma_h = 0.05$ ,  $\gamma_c = 0.5$ ) given the observed sequence of aggregate shocks from 1984 to 1998.<sup>29</sup>
4. Document the average debt-to-income ratio and the bankruptcy filing rate in the economy in 1998.

We conduct the above experiment for an economy of 10 million households and average the results over many simulations. Our main quantitative results are summarized in Table 2. We find that by explicitly modeling the dynamic learning process of households and creditors in an environment with aggregate uncertainty, our model is able to account for 58% of the rise in the consumer debt-to-income ratio and 64% of the increase in the bankruptcy filing rate over this period.

Table 2: Quantitative Results

	Data	Model
Debt-to-Income Ratio in 1998 (%)	9.1	7.1
Bankruptcy Filing Rate in 1998 (per 1,000)	8.3	6.0

In what follows, we evaluate the robustness of our results to alternative choices of the gain parameters  $\gamma_h$  and  $\gamma_c$ . We also analyze the relative importance of household and creditor learning in generating the observed rise in debt and the bankruptcy filing rate. Finally, we compare the performance of our model to close alternatives that abstract from learning.

### 5.3 Sensitivity Analysis and Decomposition

Since the existing literature provides relatively little guidance regarding the choice of the gain parameters  $\gamma_h$  and  $\gamma_c$ , we perform a sensitivity analysis to quantify the

<sup>28</sup>This implies that the beliefs of both households and creditors in 1983 are still consistent with the true data generating process.

<sup>29</sup>In this part of the experiment, creditors update their beliefs based on realized default rates, while households update their beliefs based on the realized transitions of the aggregate state.

effects of changes in these parameters on our model’s predictions. Exploring the sensitivity of our results to changes in these parameters also allows us to decompose the effects of learning by households and creditors. We repeat our experiment for various values of  $\gamma_h$  and  $\gamma_c$  and report the results in Tables 3 and 4.

Table 3: Varying  $\gamma_h$ , Holding  $\gamma_c = 0.50$

	Data	0.00	0.01	0.02	0.05
Debt-to-Income Ratio in 1998 (%)	9.1	6.1	6.4	6.6	7.1
Bankruptcy Filing Rate in 1998 (per 1,000)	8.3	5.7	5.4	5.5	6.0

Table 4: Varying  $\gamma_c$ , Holding  $\gamma_h = 0.05$

	Data	0.00	0.05	0.30	0.50
Debt-to-Income Ratio in 1998 (%)	9.1	5.8	6.7	7.1	7.1
Bankruptcy Filing Rate in 1998 (per 1,000)	8.3	7.9	3.6	4.9	6.0

Clearly, the results presented in Tables 3 and 4 imply that the choice of gain parameters has a large effect on the quantitative predictions of our calibrated model. However, our model is still able to generate a large increase in both the debt-to-income ratio and bankruptcy filing rate for relatively small values of  $\gamma_h$  and  $\gamma_c$ . To see this, note that when  $\gamma_h = \gamma_c = 0.05$  (Table 4), the model captures 43% of the rise in the debt-to-income ratio and 25% of the rise in the bankruptcy filing rate. This gives us confidence that, even though our results are sensitive to the choice of these parameters, the learning-driven credit channel that we emphasize here is in fact quantitatively significant in explaining these facts.

It is also evident that, for the case in which  $\gamma_c$  and  $\gamma_h$  are strictly positive, the debt-to-income ratio is more sensitive to changes in  $\gamma_h$  than  $\gamma_c$ . To see this more clearly, note that a reduction in  $\gamma_c$  from 0.50 to 0.05 (a 90% reduction) reduces the debt-to-income ratio from 7.1% to 6.7%, whereas a reduction in  $\gamma_h$  from 0.05 to 0.02 (a 60% reduction) reduces the debt-to-income ratio even further. Recall that  $\gamma_h$  affects the rate at which households change their beliefs about the aggregate state transition matrix, which in turn directly affects the household’s precautionary savings motive, and ultimately their incentives to borrow. Changes in  $\gamma_c$ , on the



other hand, indirectly affect households' incentives to borrow through bond prices. Hence, it is intuitive that the debt-to-income ratio is more sensitive to changes in  $\gamma_h$ .

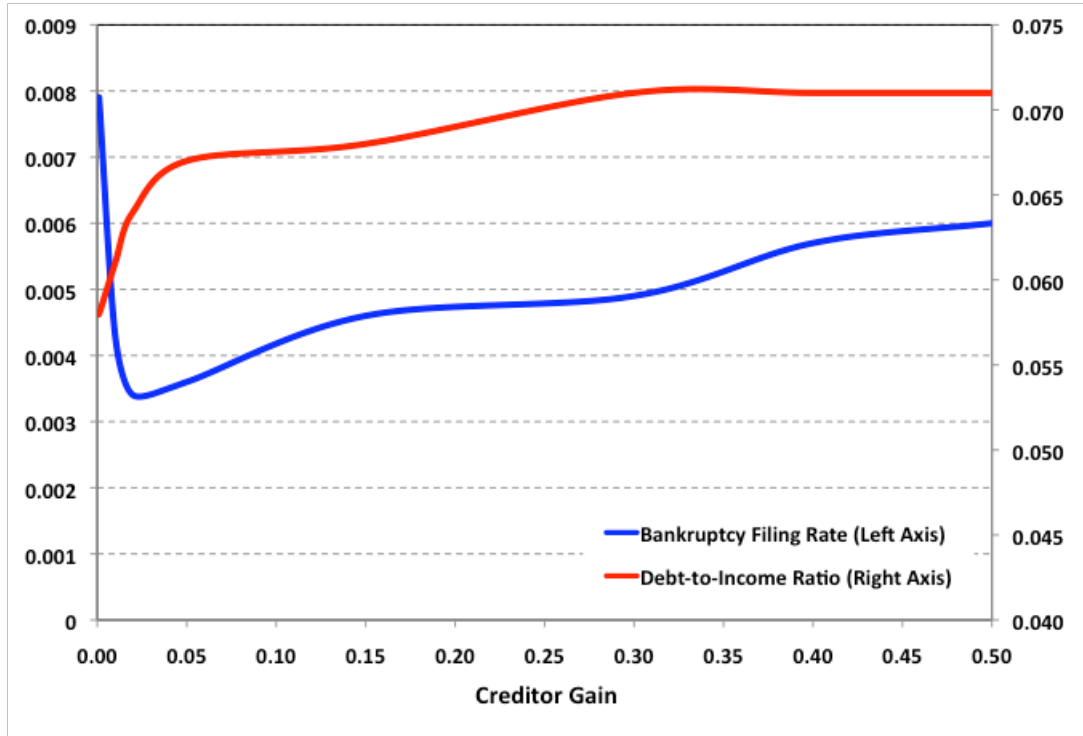
The opposite holds true for the bankruptcy filing rate, which is more sensitive to changes in  $\gamma_c$  than  $\gamma_h$ . For example, changing  $\gamma_h$  from 0.05 to 0.02 (a 60% reduction) reduces the bankruptcy filing rate from 6.0 to 5.5 per 1,000, while decreasing  $\gamma_c$  from 0.50 to 0.30 (a 40% reduction) decreases the default rate from 6.0 to 4.9 per 1,000. To understand why this is, notice that varying  $\gamma_c$  affects the rate at which creditors change their beliefs about default probabilities. Since creditor beliefs directly affect interest rates, we can conclude that the value of  $\gamma_c$  varies how sensitive interest rates are to the realized sequence of aggregate shocks. From the household's perspective, changes in interest rates make repaying debt either more or less attractive relative to filing for bankruptcy, and hence affect their default decision at the margin. This fact implies that the bankruptcy filing rate should be more sensitive to changes in the degree of creditor learning.

Interestingly, we find that while the debt-to-income ratio is a monotone function of the gain parameter, the bankruptcy filing rate exhibits a sharp monotonicity. This is best seen by looking at Figure 3, where we plot the response of the debt-to-income ratio and bankruptcy filing rate to changes in the rate of learning by creditors ( $\gamma_c$ ), holding the rate of learning by households fixed ( $\gamma_h = 0.05$ ). From the figure, we can see that the bankruptcy filing rate is first decreasing and then increasing as we increase the rate of creditor learning.

The reason for this non-monotonicity is the following: Consider a model with household learning ( $\gamma_h > 0$ ) and without creditor learning ( $\gamma_c = 0$ ). In response to a sequence of favorable aggregate shocks, households will begin to discount the probability of transitioning to an adverse state in which the income process has a larger variance and lower mean. Everything else constant, this raises the value of default relative to repaying since the punishment – particularly the inability to borrow to smooth consumption – is perceived to be less painful by households. Thus, we would expect to find the bankruptcy filing rate to be an increasing function of  $\gamma_h$  when  $\gamma_c = 0$ .

Now consider the case with household learning ( $\gamma_h > 0$ ) and creditor learning ( $\gamma_c > 0$ ). As  $\gamma_c$  increases, interest rates will fall in response to a sequence of favorable aggregate shocks, thus raising the value of repaying relative to defaulting for any given level of debt since households will be able to borrow on better terms. Yet a decrease in interest rates also induces households to take on more debt. Since default rates are increasing in a household's debt level, we should expect this effect to cause the incidence of default, or the bankruptcy filing rate, to rise. The relevant question then, is which effect is stronger? From Figure 3 we can infer that the effect of lower

Figure 3. The Effect of Creditor Learning ( $\gamma_h = 0.05$ )



interest rates on households' default decisions dominates for relatively low rates of creditor learning ( $\gamma_c < 0.02$ ), while its effect on households' incentives to borrow win out for relatively high rates of creditor learning ( $\gamma_c > 0.02$ ). Hence, we find that the bankruptcy filing rate is a non-monotone function of  $\gamma_c$ .

## 5.4 Evaluating Alternative Explanations

In this section we compare the quantitative implications of our model with two close alternatives, each of which is intended to capture a competing view of the Great Moderation. The key difference that we would like to emphasize between our model and the two alternatives that we consider here is that while ours allows agents to update their beliefs in response to changes in economic fundamentals, both alternatives abstract from learning.

The first alternative that we consider is based on the view that the Great Moderation represented a structural break (SB) in the underlying parameters of the true data generating process. In particular, one can think of this view being manifested

by a one-time change in the aggregate state transition matrix in 1984, resulting in a new data generating process which is characterized by lower aggregate volatility. While there are many ways to model this change, we assume that this structural break is unforeseen by the agents in the model. Moreover, we consider the case in which households and creditors learn of the new transition matrix prior to making any decisions in 1984. Finally, households and creditors believe that this change is permanent and irreversible.

A competing view of the Great Moderation is that it was simply a sequence of lucky draws (LD) from an unchanged data generating process. We choose to model this view by assuming that households and creditors are endowed with beliefs that are consistent with the true data generating process (the pre-1984 process) and solve their problems accordingly in response to the realized sequence of aggregate shocks. Note that this corresponds to the special case of our model in which we shut-down both household and creditor learning ( $\gamma_h = \gamma_c = 0$ ).

The debt-to-income ratio and bankruptcy filing rate generated by each of these alternatives are presented in Table 5. While the first alternative overshoots and the second undershoots the debt-to-income ratio in the data, neither alternative is able to match the performance of our model in replicating the increase in the bankruptcy filing rate observed in the data.

Table 5: Evaluating Alternative Explanations

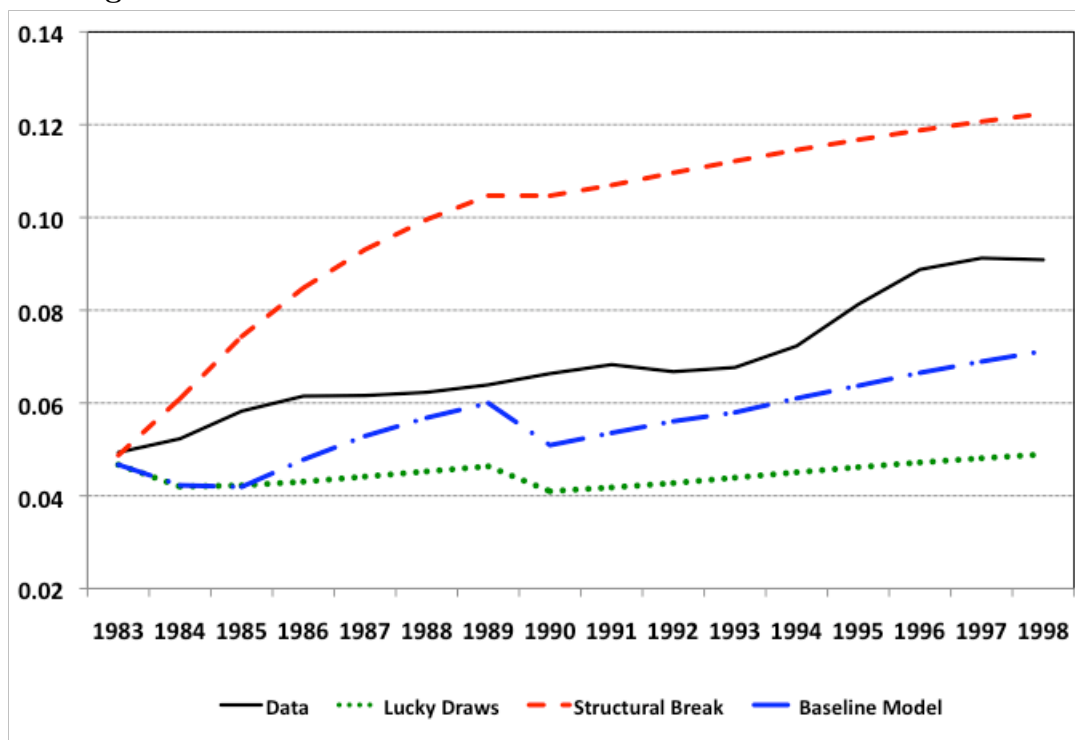
	Data	Our Model	SB	LD
Debt-to-Income Ratio in 1998 (%)	9.1	7.1	12.2	4.9
Bankruptcy Filing Rate in 1998 (per 1,000)	8.3	6.0	5.4	2.2

The deficiencies of these alternative models are even more evident when we look at the time series of the debt-to-income ratio implied by each model depicted in Figure 4. The rate of increase in the debt-to-income ratio of the structural break alternative is far more rapid than we observe in the data, while the debt-to-income ratio hardly moves at all for the lucky draws alternative. Perhaps most importantly, the structural break alternative leaves no room for other factors that have been discussed in the literature as potential causes of the rise in consumer debt, such as those emphasized by Livshits et al. (2010). The lucky draws alternative, on the other hand, clearly has little if any ability to explain this fact.

Further evidence against the structural break model is found in the historically high profits earned by credit card companies during this period despite the rising con-

sumer bankruptcy rate (see Ausubel (1997)). The structural break model assumes that creditors formed their expectations during this period based on the true default rates implied by the post-1984 stochastic process. Therefore, creditors in this model earn zero profits on average. Our model with learning, on the other hand, is consistent with this finding. To see this, recall that in the presence of learning, creditor expectations of default rates during this period exceeded the realized default rates. This implies that creditors demanded default premia above what was necessary to cover ex-post losses from default, leading to large profits.

**Figure 4. Household Debt-to-Income Ratios: Simulation Results**



Together, these results suggest that a model which takes into account the presence of aggregate uncertainty but abstracts from learning and changes in other key model parameters (such as the cost of filing for bankruptcy, or the transaction cost associated with extending credit to households) will be unable to account for the simultaneous rise in consumer debt and bankruptcies that took place during the Great Moderation.

## 6 Conclusion

Is learning by households and creditors important for explaining the simultaneous rise in the consumer debt-to-income ratio and bankruptcy filing rate during the Great Moderation? The analysis of this paper suggests that it is. In this paper we develop a model of optimal default in which households and creditors learn about economic fundamentals in the presence of aggregate uncertainty. We show that in response to a sequence of favorable aggregate shocks, similar to those realized during the Great Moderation, households begin to discount the probability of transitioning to recession and creditors reduce their default expectations. As a result, households' precautionary savings motive is reduced and interest rates fall, both of which lead to increased household borrowing and incidence of default. We find that a calibrated version of our model is able to capture 58% of the rise in the unsecured consumer debt-to-income ratio and 64% of the increase in the bankruptcy filing rate observed in the data. We also demonstrate that neither household nor creditor learning on its own is sufficient to explain the observed trends in the data – both are quantitatively important mechanisms. Finally, we illustrate that close alternatives to our model that abstract from learning generate results that are at odds with the data.

The fact that our model is unable to fully account for the rise in the debt-to-income ratio and the bankruptcy filing rate over this period suggests a prominent role remains for the mechanisms emphasized by Livshits et al. (2010). Thus, while we conclude that learning in the presence of aggregate uncertainty can account for roughly half of the rise in both of these statistics, we recognize that the unexplained portion is likely due to other factors that have already been emphasized in the literature. We therefore view the learning-driven credit channel developed in this paper as complementary to the existing body of work that endeavors to explain these facts.

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## Appendix A: Proofs

**Lemma 4.2:** We show that  $T_q$  is a contraction mapping by proving that it satisfies Blackwell's sufficient conditions for a contraction.<sup>30</sup>

- **Monotonicity:** Suppose  $W^G(b, y; s) \leq V^G(b, y; s)$  for all  $\{b, y, s\}$ . Then:

$$\begin{aligned} (T_q V^G)(b, y; s) &= \max \left\{ V^D(y; s), \max_{b'} \left\{ u(y + b - q(b', y; s)b') + \beta \mathbb{E} [V^G(b', y'; s')] \right\} \right\} \\ &\geq \max \left\{ V^D(y; s), \max_{b'} \left\{ u(y + b - q(b', y; s)b') + \beta \mathbb{E} [W^G(b', y'; s')] \right\} \right\} \\ &= (T_q W^G)(b, y; s) \end{aligned}$$

- **Discounting:** Let  $a \in \mathbb{R}_+$ . Then

$$\begin{aligned} (T_q(V^G+a))(b, y; s) - (T_q V^G)(b, y; s) &= \max \{ V^D(y; s), \max_{b'} \{ u(y+b-q(b', y; s)b') \\ &\quad + \beta \mathbb{E} [V^G(b', y'; s') + a] \} \} - (T_q V^G)(b, y; s) \\ &\leq \max \{ V^D(y; s), \max_{b'} \{ u(y+b-q(b', y; s)b') \\ &\quad + \beta \mathbb{E} [V^G(b', y'; s')] \} \} + \beta a - (T_q V^G)(b, y; s) \\ &= \beta a \end{aligned}$$

Thus, the operator is a contraction mapping, and there exists a unique fixed point by the contraction mapping theorem. Denote the fixed point associated with the operator  $T_q$  as  $V_q^*(b, y; s)$ .

**Theorem 4.3:** We will show that if  $q_{t+1}(b, y; s) \geq q_t(b, y; s)$  for all  $(b, y; s)$  then  $V_{q_{t+1}}^*(b, y; s) \geq V_{q_t}^*(b, y; s)$ , i.e. that the fixed point under the  $T_{q_{t+1}}$  operator is at least as large as the fixed point under the  $T_{q_t}$  operator for the entire state space. Since the value of default is invariant to the bond price schedule, this is equivalent to showing that the value of not defaulting is at least as large under  $q_{t+1}$  as under  $q_t$  for the entire state space.

Let  $V_{q_t}^*(b, y; s)$  be the unique fixed point under  $q_t$  with associated policy functions  $b_{q_t}^*(b, y; s)$  and  $D_{q_t}^*(b, y; s)$ . Applying the operator under  $q_{t+1}$  to this fixed point gives us:

$$(T_{q_{t+1}} V_{q_t}^*)(b, y; s) = \max \left\{ V^D(y; s), \max_{b'} \left\{ u(y + b - q_{t+1}(b', y; s)b') + \beta \mathbb{E} [V_{q_t}^*(b', y'; s')] \right\} \right\}$$

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<sup>30</sup>In the appendix we drop the  $t$  subscripts on the value functions and decision rules for ease of notation.

$$\begin{aligned}
&\geq \max\{V^D(y; s), u(y + b - q_{t+1}(b_{q_t}^*(b, y; s), y; s)b_{q_t}^*(b, y; s)) \\
&\quad + \beta\mathbb{E}[V_{q_t}^*(b_{q_t}^*(b, y; s), y'; s')]\} \\
&\geq \max\{V^d(y; s), u(y + b - q_t(b_{q_t}^*(b, y; s), y; s)b_{q_t}^*(b, y; s)) \\
&\quad + \beta\mathbb{E}[V_{q_t}^*(b_{q_t}^*(b, y; s), y'; s')]\} \\
&= V_{q_t}^*(b, y; s)
\end{aligned}$$

Successively applying the operator  $T_{q_{t+1}}$  gives a non-decreasing sequence of functions, all at least as large as  $V_{q_t}^*(b, y; s)$ , that converges to some limit—the fixed point under  $T_{q_{t+1}}$ :  $V_{q_{t+1}}^*(b, y; s)$ . Thus,  $V_{q_{t+1}}^*(b, y; s) \geq V_{q_t}^*(b, y; s)$ . Moreover, since  $V_{q_{t+1}}^D(y; s) = V_{q_t}^D(y; s)$  for all  $t$ , we conclude  $V_{q_{t+1}}^R(b, y; s) \geq V_{q_t}^R(b, y; s)$ . As a result,  $V_{q_{t+1}}^R(\tilde{b}_t^{y,s}, y; s) \geq V_{q_t}^R(\tilde{b}_t^{y,s}, y; s) = V^D(y; s)$ , and the debt thresholds under  $q_{t+1}$  are no greater than the thresholds under  $q_t$ .

**Theorem 4.4:** Suppose at date  $t$ ,  $s_t = s^j \in X_t(b', y)$  is realized. Then learning implies

$$\mathbb{E}_{t+1}[D(b', y'; s')|y; s] = \gamma_c DR_t(b'; s^j|y) + (1 - \gamma_c)\mathbb{E}_t[D(b', y'; s')|y; s].$$

Since  $s^j \in X_t(b', y)$ ,  $DR_t(b'; s^j|y) \leq \mathbb{E}_t[D(b', y'; s')|y; s]$ . Thus,

$$\begin{aligned}
\mathbb{E}_{t+1}[D(b', y'; s')|y; s] &= \gamma_c DR_t(b'; s^j|y) + (1 - \gamma_c)\mathbb{E}_t[D(b', y'; s')|y; s] \\
&\leq \gamma_c \mathbb{E}_t[D(b', y'; s')|y; s] + (1 - \gamma_c)\mathbb{E}_t[D(b', y'; s')|y; s].
\end{aligned}$$

Thus,  $\mathbb{E}_{t+1}[D(b', y'; s')|y; s] \leq \mathbb{E}_t[D(b', y'; s')|y; s]$ , and hence  $q_{t+1}(b, y; s) \geq q_t(b, y; s)$ .

Now suppose at date  $t$ ,  $s_t = s^j \notin X_t(b', y)$  is realized. Then learning implies

$$\mathbb{E}_{t+1}[D(b', y'; s')|y; s] = \gamma_c DR_t(b'; s^j|y) + (1 - \gamma_c)\mathbb{E}_t[D(b', y'; s')|y; s].$$

Since  $s^j \notin X_t(b', y)$ ,  $DR_t(b'; s^j|y) \geq \mathbb{E}_t[D(b', y'; s')|y; s]$ . Thus,

$$\begin{aligned}
\mathbb{E}_{t+1}[D(b', y'; s')|y; s] &= \gamma_c DR_t(b'; s^j|y) + (1 - \gamma_c)\mathbb{E}_t[D(b', y'; s')|y; s] \\
&\geq \gamma_c \mathbb{E}_t[D(b', y'; s')|y; s] + (1 - \gamma_c)\mathbb{E}_t[D(b', y'; s')|y; s].
\end{aligned}$$

Thus,  $\mathbb{E}_{t+1}[D(b', y'; s')|y; s] \geq \mathbb{E}_t[D(b', y'; s')|y; s]$ , and hence  $q_{t+1}(b, y; s) \leq q_t(b, y; s)$ .

**Theorem 4.6:** Suppose initial beliefs  $\mathbb{E}_0[D(b', y'; s')|y; s_{-1}]$  are such that there exists state  $k$  such that  $DR_0(b'; s^k|y) \leq \mathbb{E}_0[D(b', y'; s')|y; s^i]$ . Therefore,  $X_0(b', y) \neq \emptyset$ . By

Theorem 4.4, if  $s_0 \in X_0(b', y)$ ,  $\mathbb{E}_1 [D(b', y'; s')|y; s_{-1}] \leq \mathbb{E}_0 [D(b', y'; s')|y; s_{-1}]$  and  $q_1(b', y; s_{-1}) \geq q_0(b', y; s_{-1})$ .

Next, we show that if the initial set is non-empty, then it is non-empty for all  $t$ . First, note that  $\mathbb{E}_1 [D(b', y'; s')|y; s_{-1}] = \gamma_c DR_0(b'; s^k|y) + (1 - \gamma_c) \mathbb{E}_0 [D(b', y'; s')|y; s_{-1}] \geq DR_0(b'; s^k|y)$  since  $s^k \in X_0(b', y)$ . Moreover, by Theorem 4.3, the debt thresholds are decreasing over time in response to higher bond prices, so that  $DR_1(b'; s^k|y) \leq DR_0(b'; s^k|y)$ . Therefore,  $DR_1(b'; s^k|y) \leq DR_0(b'; s^k|y) \leq \mathbb{E}_1 [D(b', y'; s')|y; s_{-1}]$  so that  $X_1(b', y) \neq \emptyset$ . An analogous argument shows that if  $X_t(b', y) \neq \emptyset$  for any arbitrary  $t$ , then  $X_{t+1}(b', y) \neq \emptyset$ , and that if we continue to draw from  $X_t(b', y)$  then  $\mathbb{E}_{t+1} [D(b', y'; s')|y; s_{-1}] \leq \mathbb{E}_t [D(b', y'; s')|y; s_{-1}]$  and  $q_{t+1}(b', y; s_{-1}) \geq q_t(b', y; s_{-1})$  for all  $t$ .

Thus,  $\mathbb{E}_t [D(b', y'; s')|y; s_{-1}]$  is a non-increasing sequence bounded below by 0. By the monotone convergence theorem  $\mathbb{E}_t [D(b', y'; s')|y; s_{-1}]$  must converge to some value  $\tilde{\mathbb{E}} [D(b', y'; s')|y; s_{-1}] \geq 0$ . Similarly, since  $q_t(b', y; s_{-1})$  is a non-decreasing sequence bounded above by  $1/((1+r)(1+\tau))$ , it too must converge to some limit  $\tilde{q}(b', y; s_{-1})$ .

## Appendix B: The Mean of the Endowment Process

In order to estimate  $\mu_s$ , we first classify the years 1969 through 1991 as corresponding to either high or low average income using annual changes in the national unemployment rate. If the unemployment rate increased by more than 1.3%, then the current year is classified as low average income. If the previous year is classified as low average income, then the current year is also classified as low average income if the decrease in the unemployment rate is less than 2/3 of the increase in the previous year. All other years are classified as high average income.

We follow Storesletten et al. (2004) and construct a repeated panel from the PSID survey years 1968-1993. We extract income data, the age of the head, and the education level of the head from the PSID main family data files, along with the 1968 interview number and relationship to the head from the PSID individual data files, for all individuals across the PSID survey years 1968-1993. We then restrict our panel to include only those individuals who are members of, or are related to, a family that was included in the 1968 SRC cross-section sample. We define income to be the log of real income in 1968 dollars (deflated by the CPI) at the family level which is the sum of the head and wife's labor income, unemployment compensation, workers compensation, and help from relatives. Income attributed to the head of the household is then defined as total income divided by the number of persons in the

family unit.

We select observations on individuals in each survey year into our panel if: (1) they are in the original sample in the previous year and the following year, (2) income is positive in the previous, current, and following year, (3) income growth rate is not less than 1/20 and not larger than 20 between the previous year and the current year or between the current year and the following year, and (4) the individual's age is between 22 and 60 years in the current year.

We then perform the following regression in order to isolate fixed effects associated with aggregate income, education, and age:

$$y_{it}^h = \theta_0 + \theta_1^T \mathbf{D}(Y_t) + \theta_2^T \mathbf{x}_{it}^h + u_{it}^h,$$

where  $y_{it}^h$  is log (per capita) income (at the household level),  $\mathbf{D}(Y_t)$  is a vector of year dummy variables,  $t = 1969, \dots, 1991$ , and  $\mathbf{x}_{it}^h$  is a vector composed of age, age squared divided by 100, age cubed divided by 10,000, and years of education completed for individual  $i$  of age  $h$  at date  $t$ .<sup>31</sup>

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<sup>31</sup>This regression also identifies the idiosyncratic, uninsurable component of the income process,  $u_{it}^h$ , which Storesletten et al. (2004) use to estimate  $\rho$ ,  $\eta_e$  and  $\eta_c$ .